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ABSTRACT

This study investigated how people reason about simple mechanical devices and physical systems, and how reasoning methods and understanding of a device evolve over a period of exposure. Twelve students attending the Teachers College at Columbia University (New York) participated in the first of two experiments; and 10 students attending the same Teachers College participated in the second experiment and were randomly assigned to five dyads. The first experiment used a quantified protocol in which spontaneous hand movements were considered evidence of modeling; dramatic reductions in reaction time were evidence of rule induction. The second experiment organized subjects into problem-solving dyads. The inductive movement between analog models and number-based rules was documented for problems about gear movement. It was found that subjects rely initially upon an analog model until a satisfactory rule is induced. However, when a problem was introduced that led to failure of the rule, subjects returned to a model. Exophoric references were used as evidence that subjects were reasoning about models. Numeric expressions were taken as evidence of rule induction under the logic that the rule operates over numbers rather than gears. The strengths of each type of reasoning are discussed. Models are conceptualized as a quasi-empirical base from which to draw the basic patterns and relevant parameters for rule induction. Three figures are included. (TJH)

The Induction of Rules from Analog, Mental Models

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ABSTRACT

The inductive movement between analog models and number-based rules was documented for problems about gear movement. It was found that subjects rely initially upon an analog model until a satisfactory rule is induced. However, when a problem was introduced that led to failure of the rule, subjects returned to a model. The first experiment used a quantified protocol in which spontaneous hand movements were considered evidence of modeling; dramatic reductions in reaction time were evidence of rule induction. The second experiment organized subjects into problem-solving dyads. Exophoric references were used as evidence that subjects were reasoning about models. Numeric expressions were taken as evidence of rule induction under the logic that the rule operates over numbers rather than gears. The strengths of each type of reasoning are discussed. Models are conceptualized as a quasi-empirical base from which to draw the basic patterns and relevant parameters for rule induction.

The induction of rules from analog, mental models

Our objective is to study how people reason about simple mechanical devices and physical systems, and, how reasoning methods and understanding of a device evolve over a period of exposure. In this paper we show that people use a model initially to reason about a mechanical system but are able to induce more abstract rules after several model simulations. Further, we will show that people return to a model of the system if their abstract rules fail. Johnson-Laird (1981) states, "By reflecting on the properties of relations represented in mental models, an individual may come to acquire a higher-order knowledge of them" (Pp. 119-120). Our research is an attempt to provide empirical evidence that this transition takes place. Moreover, we provide some interpretations of why models are able to serve as a source of evidence for the process of rule induction.

Some investigators have characterized learning and reasoning in terms of inducing and applying rules (e.g., Anderson, 1983; Kieras & Bovair, 1986). Others have discussed learning and reasoning in terms of constructing and manipulating mental models (e.g., de Kleer & Brown, 1981; Johnson-Laird, 1983). A third line of research has contrasted the performance of students taught rules or productions with those taught through models (e.g., Mayer, 1981; Halasz & Moran, 1983). In the research reported here we attempt to take a step towards unifying these lines of work by showing that, at least in some cases, people reason using mental models initially until they induce a relevant rule and reason with it. This movement can be likened to the reported change from qualitative to abstract and quantitative reasoning found in the movement of novice to expert (Chi, Feltovich, & Glaser, 1981; Larkin, McDermott, Simon, & Simon, 1980) or the movement from enactive to symbolic representation in developmental studies (Bruner, 1966). Our approach is somewhat different from expert-novice and developmental studies, however, in that we attempt to characterize the strengths and weaknesses of each type of reasoning to account for the propulsion back and forth between analog models and rule-based symbolic reasoning.

An example will show the distinction between the two styles of reasoning. Imagine three gears placed side by side and connected, much like a row of quarters laid on a table. If one tries to turn the gear on the far left clockwise what will the gear on the far right do? There are several different ways to solve this problem.

One approach is to simulate the gears. This approach relies on a mental model's capacity for supporting and directing a simulation that is analogical to physical gear movement (Schwartz & Black, 1990; Shepard & Feng, 1972). Following this method, one might first attempt to model the movement of just two connected gears where the gear on the left turns clockwise. This phase of the modeling is what deKleer & Brown (1981) termed *envisionment.* The simulation is directed at discovering what the interaction of the two connected gears will be. It is quite specific in that it begins with a particular direction of movement (clockwise) and yields a specific answer for the second gear (counter-clockwise). The resulting motion of the second gear could subsequently be modeled in conjunction with the third gear. According to a pure mental model account, the culmination of learning would be a correct simulation that can iteratively communicate motion along a chain of gears.

A second approach is to rely on a rule that states that two adjacent gears turn in opposite directions. This knowledge could be used recursively to calculate that the third gear must go in the same direction as the first; no gear movement is required. This second approach is primarily symbolic. It relies more on the relationship of opposite than it does on any motion of the gears. Learning in this case would involve mapping the opposite relation onto the odd and even properties of the gear chains involved. One would learn a parity rule that states that for odd numbers of gears, the two ends turn the same direction, and, for even numbers of gears, the two ends turn oppositely. The specific rule of adjacency is generalized to the more expedient rule about the parity relations among gears (Laird, Rosenbloom, & Newell, 1986). After a number of problems, one can imagine quickly using the oddness or evenness of the gear system to solve problems with hundreds of gears. But note that the solution using the parity rule operates over symbols and their relations (numbers and their properties; e.g., seven gears = odd), whereas, the model solution operates over gears and motions. Typically, people do learn this symbolic rule and answer large problems easily. But the path to this knowledge begins with the model simulation.

People induce the simple, symbolic parity rule using the simulation as a source of "quasi-empirical" evidence that reaveals the behavior and patterns of gear movement. One might imagine that the induction of the rule is the end of the story. But, despite the efficacy of the parity rule, its induction does not signal the end of the gear models' utility. To preview the dialectic of analog models and rules, imagine that the three gears are now arranged in a closed loop or triangle so that each gear touches two others (see fig. 1). If one tries to turn the gear on the top clockwise, what will the gear in the lower-left corner do? One typical revelation involves discovering that the parity rule can be correctly applied and still give two different answers. For example, a person may consider the initial gear on the top and the target gear in the lower-left to form a two gear system, effectively ignoring the third gear in the lower-right hand corner. This is an even number of gears and yields an answer of counter-clockwise. One can also consider the gear in the lower-left to be the end of a three gear chain counting from top to lower-right to lower-left. According to the parity rule, this should give a clockwise answer as there are an odd number of gears. A good problem solver who considers alternative applications of the rule can discover these contradictory answers.

Although the parity rule reveals its own inadequacy through alternative applications to this problem, it does not provide the correct response. In fact, it does not include the correct outcome within its domain of clockwise - counter-clockwise answers. A new simulation is required to determine what is happening. The initial simulations answered the question of what would happen to an adjacent gear if the first gear were turned clockwise or counter-clockwise. The new simulation will lead to an evisionment of what will happen if two connected gears both attempt to turn the same direction. One way this might go is that a person runs the model from the top gear to the lower-right to the lower-left and back to the the top gear. Another way might be that the person figures that the top gear will turn both bottom gears counter-clockwise. In each case, the reasoner will be confronted with the problem of what two touching gears will do if they both are turning the same direction. The envisionment used to answer this will provide the needed information that the gears will jam. This knowledge starts the development of a second or improved parity rule that takes into account the openness or closedness of the gear system. This addition to the rule would state that for an odd number of gears in a closed-loop none of the gears can turn, but for an even number of gears, the original parity rule holds.

The modeling and rule-based reasoning methods each have their liabilities. Analog modeling is cumbersome for problems with large numbers of gears or complex relations, but, rules are inadequate for slightly novel problems. By way of an analogy from instruction, it would not make sense for an adult to figure out a large multiplication problem about money by manipulating piles of pennies, but, it would also be foolish to teach a child the rules of multiplication without teaching what the numbers represent. The two styles of reasoning complement and fill in each other's weaknesses.

This research will show that the slower modeling process is relied upon both for initial problem solving and as a fallback when a rule fails. Once discovered, the more efficient and more general rule is used until the limitations of its formulation are reached. Both experiments below involve problems like the two above. These gear problems have several attractive features. The first is that they are susceptible to fairly quick discovery learning. This allows for detailed observations of knowledge evolution during a meaningful, learning task. The second is that the change in reasoning methods allows for a non-problematic operationalizing of the concepts of model and rule-based reasoning. Avoiding the imbroglio of the mental model literature, we simply define model-based reasoning to occur when people utilize an analogical representation of gears. This mode of reasoning will operate over a representation of gears and their force relations. On the other hand, rule-based reasoning will not require a representation of gears. Rather, for this domain of problems, rule-based reasoning will operate over symbols and their relations.

Although we are operationally limiting the scope of mental models to the analogical, we believe the characteristics of and movement between the two sorts of reasoning can be found in a variety of knowledge domains. In the final section of the paper we conceptualize a role for each style of reasoning that can explain the interplay between models and rules.

EXPERIMENT 1

To explore the hypothesis that people make an inductive transition from modeling to rule-based reasoning, and that ideally the model remains available for novel situations, two sets of gear problems like the ones above were developed (see fig. 1). The first set involves gears arranged in a horizontal row, or open chain. The second set has the gears arranged in a circle, or closed chain, so that there are "no loose ends." Subjects were shown six problems from the open set and then six problems from the closed set. Several predictions develop from the hypothesis.

The first prediction is that there will be much analog simulation of gear movements for the first few problems in each set. This early modeling in each set corresponds to the situations where there is no rule and when the rule fails. Evidence of analog processes has typically been achieved through chronometric studies (e.g., Shepard & Cooper, 1982). In these studies, a subject mentally rotates some object, such as a letter, to determine if it is a correct or mirrored version of a target. The mental process is considered an analog of real world motion under the logic that the mental "entity" under transformation must traverse through the intermediate positions on its way to the target position just like a physical object would. That is, the image cannot be "snapped" to the correct position. Chronometric evidence of analog rotation, then, requires that reaction times should increase linearly with increasing angles of rotation just as they would if a real world object were being rotated. This chronometric paradigm, however, requires a tightly controlled experimental task where the onset of problem solving can be guaranteed. Another type of evidence for analog modeling, more suitable to our task, can be gathered by looking at subjects' spontaneous hand movements. Given these gear problems, people typically model the problems with their hands. Although analog models are generally thought of in the visual modality, the motoric system could also serve as a substratum for the concrete representations of analog simulations (Klatzky, Pellegrino, McCloskey, & Doherty, 1989; McNeill, 1987). Subjects can create analog models with their hands and use these to simulate gear behavior. The hand movements that we observe are not the same as the gestures typically discussed in communication research (e.g., Ekman & Friesen, 1972). Rather, they are the deliberately representational gestures discussed by McNeil (1987) and Bloom (1979). They do not necessarily accompany speech and they have a segmented grammar in which portions of gestures that are meaningful in their own right can be recombined with other gestural movements.

Using spontaneous hand movements as data has three advantages over other methods. First, it is a method of data collection of which subjects are unaware. It is completely unobtrusive and requires no elicitation. Second, it seems better suited to capturing the non-verbal processes involved in analogical modeling than verbal protocols. Third, and less pragmatically, these hand movements neatly straddle the boundary between physical and mental models bringing the potential importance of analog models into theoretical relief. In a sense, the hand movements are a tool that extend one's thinking into the environment (Resnick, 1987). Or reciprocally, they are a representation that brings the physical environment into one's thinking. As will be discussed in the final section, an analog simulation plays a role similar to an actual experiment or physical simulation in the reasoner's attempts to induce rules of behavior. Hand movements, by being partially mental and partially physical models, highlight the idea that we may cogitate similarly over both internal and external models.

The second prediction is that reaction times will drop precipitously after the third problem in each set. The drop in reaction times will reflect the movement to the more efficient parity rule. Earlier research on induction leads us to expect that this drop will occur after the third problem. Gick and Holyoak (1983) found that people require at least two examples from which to generalize any patterns. In this case, the third problem, once solved, will highlight the relationship of odd numbers to clockwise answers.

As a convergent pattern of evidence, subjects who show the predicted drop in gesturing activity and reaction time should spontaneously mention the parity rule in debriefing as they explain how to answer the problems. The time and effort savings of the parity rule should make subjects use it and suggest its use if they have discovered it. Further, if the reductions in reaction time and gesturing activity are due to the employment of the parity rule, those who do *not* show the predicted reductions should not know or be able to mention the parity rule. Finally, there should be a large number of erroneous "clockwise" responses on the first problem in which the gears will lock. If subjects are applying the parity rule induced from the gears in a row, a problem with three gears in a circle should lead them to figure that it is an odd number of gears, and therefore, the last gear should go the same direction (clockwise) as the first gear. If subjects are not relying on the parity rule, there is little a priori reason to suspect that they would choose "clockwise" more often than "counter-clockwise" (i.e., treating the problem as involving three gears instead of two adjacent gears).

Setup

Subjects. Twelve subjects from the Teachers College, Columbia University were recruited to participate in the experiment.

Design. Two sets of gear problems corresponding to the open and closed chain were employed. Each set had six gear problems that increased from three to eight gears. For the open chain problems the subject was read the following type of problem: "Five gears are arranged in a horizontal line, if you try to turn the gear on the furthest left clockwise, what will the gear on the furthest right do?" For the closed chain problems the question was: "Five gears are arranged in a circle, if you try to turn the gear on the top clockwise, what will the gear just to its left do?" The problems in each set were broken into two blocks, three gears to five gears and six gears to eight gears. Within each block, the order of presentation was randomized. The randomization prevented over-cueing the odd-even relationship. The separation into blocks prior to randomization ensured that the first problems did not involve too many gears.

Procedure. The subject sat in front of a video camera. The experiment sat behind a screen, out of view of the subject. The subject was read the following text: "All of the following problems involve reasoning about gears. You should assume that each gear is touching its closest neighbors. I am only interested in your answer. If you have a question about the problem I will reread it. Be sure of your answer. If you get it wrong, I will tell you so and you will have a second opportunity to solve the problem. After that I will tell you whether you are right or wrong and go on to the next problem." The problems from the first set were then presented. Prior to the second set, subjects were told that, "In the following problems, you are to assume that each gear is touching two other gears." After completing these problems the subject was requested to teach a new subject (who was not used for any other purpose in this experiment) how to get all the answers correct. This served as the debriefing.

Results

Coding Scheme

The total times spent making each of two types of hand movements were coded: rotational gestures and interaction gestures. A rotational gesture is considered evidence that a subject was modeling the circular movement of a gear. To ensure that the hand movement was circular, we required the gesture to have a minimum of a 90° movement in a circular motion. The movement could be made with either arm, a hand, or a finger. Interaction gestures are a subset of rotation gestures. Hand movements are considered an interaction gesture if the subject touched both hands and coordinated the circular movement of each with the other. These attempts at simultaneously coordinating the movement of two gear motions are taken as evidence that the subject was determining the behavior of two touching gears (e.g., "they turn inward on each other"). Other hand movements, such as pointing, moving in a line, holding the hands motionless in the air, or speech accompaniment gestures are not considered.

Analysis of Hand Movements and Response Times

First we consider those who performed above chance in both problem sets (n=7). Later, we will briefly look at how the other subjects failed due to their incorrect models, or lack, thereof.

Because there was no time limit on a problem and some subjects took a large amount of time, we make the variances more homogeneous by using the log of the total time spent solving the problem and making the two types of gestures. The time spent solving the problems, making rotation gestures, and making interaction gestures were averaged separately for each subject for the first block of three problems and the second block of three problems for each problem set. The resulting twelve measures were dependent variables (2 sets x 2 blocks x 3 measures) in a repeated measures analysis with problem set and block of problems as within subject factors. The main findings are well captured by figure 2. Using the simpler univariate results, the effect of the block of problems on reaction time ($F(1,6)=74.4$, $p < .01$) and rotational gestures ($F(1,6)=14.5$, $p < .01$) is quite strong indicating that modeling occurred primarily for the first three problems of each set. The pattern of results for the hand movements also holds if we consider number of gestures per second.

The two way interaction of block and problem set tests whether the patterns of reduced times after the third problem is different for the open and closed gear chains. The interaction was not significant for reaction time ($F(1,6)=1.3$) or rotational gestures ($F(1,6)=.6$). However, the interaction gestures do show a different pattern for each problem set ($F(1,6)=5.2$, $p = .06$) with the most interaction gestures occurring in the first three problems of the closed chain set.

Error Analysis

As mentioned previously, overall, seven of the twelve subjects were able to solve the problems at an above chance rate for each of the two sets. Of these seven, six explicitly mentioned the parity rule in the debriefing.

Nine subjects were able to work the open chain problems successfully. These nine subjects can be used to test for the incorrect application of the parity rule for the first closed-chain problems that lock. Of these nine, four made the predicted mistake of saying "clockwise" for the first lock problem. Two others made this mistake on the second lock prob'em rather than the first. Other than these mistakes and the perseveration of a clockwise response for those subjects who performed below chance (discussed below), errors were minimal and haphazard.

Analysis of Sub-Criterion Performance

Although the sample sizes for the error data and the subjects who did not follow the idealized sequence are too small for a statistical analysis, they are revealing. The one subject who solved both sets of problems successfully but did not mention a rule in the debriefing had increasing gestures and reaction times for the second block of problems in each set. This is in accordance with the prediction that if a subject does not induce the parity rule, it will be necessary to simulate the problems with the larger numbers of gears, and, hence take more time.

The five subjects who did not perform the task to criterion fell into two groups. The first group consists of three subjects who developed an inappropriate model. Two of these thought that "the gears should turn in the same direction like tires on a truck." They both gave answers of clockwise to all the problems. The third modeled the gears as though stacked on top of each other. She deduced a rule for the first set of problems using the experimenter's feedback and a process of elimination. However, as "jamming" was not available in the problem statements of the closed chain and she had an incorrect model, she was unable to discover that the gears could lock. As such, she never solved any of the locking problems correctly.

The second group consists of two subjects who followed the idealized pattern for the first set of problems but became fixed on the recently induced rule. All of their answers to the problems with the gears arranged in a loop were based on the original rule. Their answers were incorrect for the odd problems of the second set. As revealed by the limited hand movements relative to the first set of problems, these two did not return to a model once they began making erroneous answers.

Discussion

Taken as a whole, the gestural and reaction time data of the subjects who performed to criterion provide a good fit to the hypothesis that modeling will occur prior to rule induction and as a fallback when the rule fails. Contrasting these subjects with those who did not follow the hypothesized sequence provides a fairly good fit to the implications of the strengths of model- and rule-based reasoning.

For the successful subjects, analog modeling, as documented by circular hand movements, occurred during the very first problems where one would assume there were no rules in place. Those who began with an incorrect model were unable to move forward in their learning. For the successful subjects, modeling also took place for the first problems of the closed-chain set where the error patterns confirm that rule application was ineffective. On the other hand, subjects who did not model the closed chain problems were unable to discover that the gears could lock. They fixed on the previous outcomes of the parity rule. Looking at the rule side of the two reasoning methods, the sharp decrease in reaction times after the third problem is what would be expected if subjects had indeed induced a rule. The effect of not inducing a rule can be seen with the one subject who had steadily increasing reaction times and gestures as the number of gears increased. This subject restarted over and over as he lost his place iterating clockwise - counter-clockwise along the chains of gears.

The interaction gestures are of interest as they provide some evidence of modeling being used for different purposes. de Kleer & Brown (1981) have argued that *envisionment* of the basic causal relations in a model is a necessary precursor for a successful simulation of the system (cf. diSessa, 1983). In this case, the interaction gestures are a simulation directed at discovering that the gears force each other in opposite directions, or, that they will lock if moving in the same direction. This is different from the simulation process involving the rotation gestures that communicate the results of one pair of gears to the next. Unlike the rotation gestures, the interaction gestures occurred somewhat on the first problems of the first set but more so on the first problems of the second set. This is to be expected. For the problems in which the gears lock, it is the basic causal relationship between two gears that is in question. Subjects relied on an envisionment simulation to confirm that the gears have to turn oppositely or not at all.

EXPERIMENT 2

The first experiment is most revealing about the model-based reasoning or lack thereof. The second experiment is designed to get a closer look at the rule-based reasoning. Unlike analog simulations, rule-based reasoning is readily susceptible to verbal formulation. In this experiment we gather verbal evidence for the movement between the two forms of reasoning.

It is our hypothesis that people will use more quantitative language after inducing the parity rule than prior to its induction. This will reflect the change in the referents of their reasoning from gears to number. On the other hand, prior to the induction of the rule, subjects will discourse over the models of gears they are creating. If this is the case, exophoric references to objects and motions that are available in the visual space but not in the sentential context will be highest before the induction of the rule. To confirm that these exophoric references are to the models, there should be a strong correlation between exophoric references and rotational gestures.

Like much short term discovery-based instruction, the first experiment suffered somewhat from subjects' failures to induce the correct rules. To improve the learning, we made two changes.

First, the problems were presented in strictly ascending order with the three-gear problem first, the four-gear problem second, and so forth. It was felt that this would highlight the relationship between the alternating clockwise and counter-clockwise answers with the odd and even pattern of the number of gears.

Second, subjects worked in pairs as they attempted to solve the problems. The reasons we expected pairs of problem solvers to do better than individuals will be described elsewhere (Schwartz & Black, in preparation). For the purposes of the second experiment, it is enough to note that in the first experiment over 50% of the subjects were able to solve the problems above chance. This gives a good probability that in any dyad, at least one participant will figure things out.

Using dyads has an added benefit in that it provides an excellent method of eliciting the verbal protocols required for the analysis of language use. Unlike the speech of think-aloud protocols, it is a natural mode of expression and removes the experimenter from the loop.

The final modification of the first experiment is designed to counter the potential objection that subjects were not using the model as a fallback in the closed-loop problems. It might be argued that the closed-loop problems were a completely new set of problems and did not have any commerce with the problems in which the gears were arranged in a line. According to this objection, the modeling at the start of the closed-loop problems is not due to the lacking of the rule from the first set but rather is due to confronting a totally new problem. To show that this is not the case, we add two slightly changed problems to each set. The initial gear turns counter-clockwise instead of clockwise for one of these problems, and, the initial gear is on the opposite side of the configuration for the other problem. For gears in a circle, this places the initial gear at the bottom of the circle rather than the top.

One of the strengths of a mental model is that it provides a method for determining which problem variables are relevant to include in the construction of a rule. The result of changing the direction of motion or handedness of the problem is not contained in the original parity rule. The parity rule induced from the first problems should be overly specific to an initial gear on the right turning clockwise. Accordingly, reaction times should rise for the first exposure to these two new problems as subjects attempt to determine whether these are relevant parameters. This will provide a replication on a smaller scale of the fallback modeling found for the closed-loop problems. Further, if the rule from the first set of problems is carried over and incorporated into the second set of problems, subjects will not need to model the modified problems for the second set. They will have already generalized their parity rule as to initial gear motion and location. One might think of the generalization of direction as going from the following partial expression of the rule:

> *If there is an odd number of gears and the first gear is going clockwise, then the last gear will go clockwise.*

To a more general rule:

> *If there is an odd number of gears, then the last gear will go the same direction as the first gear.*

Once this generalization is in place, it should be incorporated into the second parity rule about gears in a closed chain. If subjects do not remodel on the second exposure to the modified problems, this will suggest that the first and second set of problems are related.

Setup

Subjects. Ten subjects, randomly assigned to five dyads, participated in the experiment as partial fulfillment of a Teachers College course.

Design and procedure. The materials and procedures for this experiment are the same as the first with four modifications. a) Subjects work in tandemn, facing each other. b) The problems are presented in the ascending order of the number of gears involved. c) A nine and a ten gear chain are added to each set of problems. The nine-gear problem is modified from the prior problems so that the initial gear is said to turn counter-clockwise. The ten-gear problem moves the initial gear to the opposite side of of the gear configuration from the prior problems. d) There is no debriefing session.

Results

Coding Scheme

The unit of analysis is the dyad and not the subject. Much of the time spent solving these problems was spent creating a common representation of the problem. For example, subjects would negotiate a representation of the gears that would allow the oppositely facing parties to refer to the same gear as "right" or the same movement as "clockwise" (Clark & Marshall, 1981). Once this knowledge space was created there was a tremendous amount of language overlap and knowledge sharing. A subject often started an expression while the partner finished it. Additionally, members of the dyad would alternately take charge of expressing the reasoning they were sharing.

Protocols were coded for: (a) the problem during which the parity rule was stated (to show that the rule was indeed discovered during the course of the problem solving and to be used in subsequent analyses as a demarcation point between pre- and post-rule induction); (b) problem-solving duration (to see if the response times dropped upon induction of the rule); (c) the occurrence of quantitative and exophoric expressions (to see if the object of reasoning changes from the gear models to numbers after the parity rule is induced); and, (d) initiations of rotational gestures (to test whether exophoric references were made with respect to the analog gear representations).

We consider the dyad to have induced a parity rule when they explicitly mention the words "odd" or "even" in conjunction with a statement about the answer space of the problem. This includes expressions like "all the odd ones go clockwise..." or "even problems go the opposite way..."

An expression or word was considered quantitative if it involved a numerical concept. This included cardinal numbers, ordinal numbers such as "the first one," the words "odd" and "even", and phrases like "Let's count," and "How many?" It is our contention that these expressions reflect an emphasis on the numerosity of the gears as would be expected if subjects were reasoning about the odd and even properties of the gears. Exophoric references were coded according to whether the reference was unambiguously a selective exophoric demonstrative reference as described by Halliday and Hasan (1976). This includes the words "this," "these", "those", "that", "here", and "there." It was required that the reference be external to the sentential context. Thus an expression like "this goes that way" contains two exophoric references, but, an expression like "that problem is the one we did before" contains no exophoric references. These expressions refer to the visible context and reflect subjects' reasoning over the analog models they had gesturally constructed. This method of coding is highly conservative. Given that the gestures were highly salient within the conversational frame, it would have been reasonable to include the substitutive expression "the one" as an exophoric reference. However, to avoid possible confusions with the number one, this expression was not coded. Further, expressions that contain locative information, such as "the gear on the far left," are good indicators of reference to a spatial model. However, as we are not discussing the spatial properties of these models in this paper, we do not include locative references.

Analysis

If we consider the open and closed problem sets as separate attempts to induce a rule, we have ten problem sets altogether (5 dyads x 2 problem sets). In nine of these ten problem sets there was an explicit and unambiguous mention of the parity rule. For the open chain problems this occurred on average during or just before the fourth problem (six gears). For the second set, the parity rule that included the jamming outcome was discovered on average at the end of the second problem (four gears). In only one of the nine problem sets was the rule stated prior to the third problem. This occurred after three minutes into the first problem of the closed chain.

For the four dyads that stated the parity rule in the closed chain set, there was an average eight-fold decrease in reaction times after the statement of the parity rule ($F(1,3)=25.5$, $p < .02$). We use this to infer that the dyad that did not explicitly mention the parity rule for the closed set had actually induced it between the fifth and sixth problems. The drop in reaction times between the two problems was from 192 seconds to 17 seconds.

The number of exophoric and quantitative usages were dependent measures in a repeated measures analysis. Their number of occurrences before and after the rule statement in each of the two problem sets created eight dependent measures (2 measures x 2 blocks *(before & after)* x 2 sets). The two way interaction between the before & after rule factor and the type of expression proved to be quite reliable given the sample size ($F(1,4)=7.24$, $p < .055$). This means that dyads used exophoric references prior to the induction of the rule and quantitative expressions after the induction of the rule. There is no three-way interaction with the type of problem set ($F(1,4)=.01$) indicating that the pattern of usage for the two types of expressions was the same for both problem sets. Figure 3 summarizes these results by showing the average number of expressions per second for each problem set.

As a manipulation check, we coded the number of rotational gestures initiated within the phrase boundaries of both exophoric references and quantitative expressions. If exophoric references were referring to the model, there should be a high correlation between the number of references and gestures made within each phrase boundary. Further, if quantitative expressions were not primarily with reference to a gear model, they should have low correlation with the hand movements. Averaging the correlations of the exophoric references with the rotational gestures for each of the five dyads, there is a high degree of association given the coarseness of the measures ($R=.47$). For the quantified phrases, the correlation is quite low ($R=.08$). This pattern held for all five dyads. Thus, the exophoric references were made with respect to the model, whereas, the quantitative expressions were not.

The low correlation of the quantitative expressions with the rotational gestures helps dispel a potential confound. It could be reasonably argued that the increase in quantitative expressions reflected a more precise method of referring to a gear in the larger chains. An expression such as "the fourth one" could be a more effective reference with large collections of gears than saying "this one." The low correlation, however, shows that the quantitative expressions were not taking over the referential role of the exophoric references.

Given the high correlation of reaction time and time spent gesturing in the first experiment, we simplify the analysis of the effect of changing the direction and position of the initial gear by using reaction times. The effect was in the predicted direction of increased times relative to the immediately preceding problem for the first set but showed little increase in the second set. In the open chain, the nine and ten problems had a mean reaction time of 39.4 seconds, whereas, the immediately preceding problem of eight gears had an average reaction time of 5.4 seconds. On the other hand, for the closed chain, the nine and ten problems averaged 9.0 seconds and the preceding problem took 5.0 seconds to answer. While this pattern is in the predicted direction, the interaction did not reach significance as only four of the dyads followed this pattern.

Two further patterns in the data are worth noting. In an even stronger fashion than in the first experiment, errors (6 total) are confined to the first two lock problems. As with experiment 1, this is interpreted as an over-reliance on the rule.

The second pattern provides some confirmatory evidence for the interpretation of the interaction gestures in the first problems of the closed chain set. In experiment 1, it was found that interaction gestures were significantly more prevalent for these problems. In this experiment, we use a rough measure that shows the same behavior. Coding for the word "touch," it was found that it occurred on the average 4.2 times for the first lock problem and 2.2 times for the second problem in the set. Excluding these two problems, it occurs with a frequency of .3 times per problem overall. This pattern holds for four out of five pairs with the fifth dyad never using the word "touch". The use of the word "touching" can be interpreted as the subjects attempting to understand exactly how two connected gears moving in the same direction can behave. Like the interaction gestures of the first experiment, this suggests that subjects return to the model with a specific question about gear behavior.

Discussion

Unlike experiment one, the transition to the parity rule is explicitly available through subjects' own attempts to explain their reasoning to their partners. That this does not occur on the first problem and that reaction times drop sharply after its mention are strong evidence that these subjects actually learned the rule rather than became more proficient and tacit at modeling the problem. The fact that this occured around the third problem backs up our use of the third problem as the probable point of induction in the first experiment. It lends furthers credence to the idea that the drops in reaction time and gesturing activity in experiment one were due to the rule's induction.

The switch from exophoric references to quantitative expressions shows that subjects were relying on number rather than gears once the rule was induced. The replication of this pattern for both the open and closed chain, as in the case of the first experiment, also provides some insight into the limitations of rules and the strengths of models for handling novel problems as will be discussed next.

SUMMARY

We have shown through a variety of data that people initially rely on analog simulations for problems about mechanical systems and that the models underlying the simulations are an important precursor to more symbolic, rule-based reasoning. They provide a place from which to start building rules. However, models are quickly left if a more expedient rule is induced. The rule leads to faster solutions, a type of fixedness for novel problems, and, potentially, the recognition of its own limitations through contradiction detection. The abstract knowledge incorporated into the rule, however, is not enough to repair itself. Detecting that something is amiss does not provide the knowledge of how to fix it. Subjects returned to a model in an attempt to figure out what exactly was happening. Once they were able to run the simulation in such a way as to see that the gears would lock, they improved the parity rule to handle the new, closed loop configuration of gears.

Similarly, when a new and potentially important factor is introduced into the problem space, the model is used to see if this variable should be added to the parameters used for reacning a solution. Interestingly, there was some evidence that this process of returning to the model is important for the generalization of the rule. Although the small sample size and single task prevent definitive proof of this point, subjects returned to the model to determine that the direction of motion did not undermine the basic odd/even pattern that controlled the outcome of the gears. From this return to the model, they were able to generalize the rule to either type of motion and a simple same/opposite decision. This generalization was already incorporated into the second parity rule by the time subjects answered another problem that changed the direction of the initial gear.

GENERAL DISCUSSION

To this point we have documented the movement between models and rules. In this section we will provide a way to conceptualize the strengths and weaknesses of each type of reasoning that can explain the motivations for the dialectic. The strengths of rules are much like those of any formal or abstract system of thought. They provide a method of distilling, framing, and solving problems in the most expedient manner. They allow for advancements in knowledge by making certain relationships more readily apparent, such as contradiction or oddness and evenness. They also are capable of generalization beyond the particulars of the problems that give rise to them.

Models, on the other hand, are central in exactly those situations where rules have no answer within themselves. To account for this, we are positing that a model is critical in the process of creating and testing hypotheses about possible input and output parameters for a rule (Holland, Holyoak, Nisbett, & Thagard, 1986; Levine, 1975). One can think of a rule as a computational function that takes certain values as input and returns a value. The process of rule induction is an attempt to determine both the number of variables and the domains of these input and output parameters. A model, much like a physical simulation, generates phenomena for the processes of induction to work over. This can be most clearly seen with the modeling that occurs for the first closed chain problems.

Creating an idealized sequence, we can imagine that at the beginning of the closed chain problems subjects have induced a function from the first set of problems that takes the direction of the initial gear and the number of gears as input parameters with direction as the output domain. For these closed-loop problems, there is a new potential variable, configuration or open-closedness. Subjects must determine if this variable should be included in the function's input parameters. Once they detect that the function is misapplied for the closed configuration, they can include a variable of the domain open or closed into the function's formulation. A second critical issue is what the output of the function should be. The first parity rule has an output domain of direction. The crucial step in inducing the new parity rule is discovering that the domain for the output should include jamming within its space of possible outcomes. There is no way to discover this new domain of outcomes within the function. Fortunately, unlike rules, a model is not constrained by its abstract representation to a limited set of inputs and outcomes. In a very real sense, a model is a thought experiment that provides the raw data on which a rule is built. Subjects return to the model to discover what other phenomena can be included within the domain of the function's output. Once this is accomplished they can then discover the pattern that controls the output of the function.

Given the view that models provide a quasi-empirical space from which we determine possible behaviors and their patterns, it is important to make some positive claims about why they have these unique capabilities. Putting the question another way: Why is it the case that the knowledge of gear behavior exerts itself through the model? Why can't the knowledge be used simply as a set of rules without the mediating referential representation? If we limit ourselves to analog models, one answer is that this knowledge of change appears to have a special relationship with deep experiential knowledge.

There is anecdotal and empirical evidence that people rely on imagery for novel problems and in periods of confusion (Sheehan & Lewis, 1974). This supports our contention that analog models are used for novel problems or in times of rule failure. A typical account of this reliance on imagery is that imagery provides a holistic view of the situation (e.g, Kaufmann, 1979). A second possible account is that images have special working memory and long term memorial properties (Baddeley, 1986; Shepard, 1967). A third account might be that they have representational properties that reduce cognitive load by removing the need to explicitly recalculate Euclidean relationships during spatial transformation (Pinker & Kosslyn, 1978). We would like to argue that an analog model additionally provides access to a basic knowledge of force dynamics that may not be available to more verbal and symbolic processes. This follows Shepard's (1984) lead, that analog models have implicitly incorporated the constraints of the physical world. For these gear problems, this constraint might be roughly formulated as: *A moving object tends to force a touching object in the same direction.*

There are two intertwined reasons for believing analog models have privileged access to this knowledge. The first reason is that the experiential linkages of an analog representation may have a priming effect on our implicit knowledge of the world. The second reason is that primitive knowledge of physical change may only be able to exert itself dynamically in a fashion that is an analog of the combined experiences that created it. Because the experiences that give rise to this knowledge are dynamic, the knowledge requires representations of objects to manipulate dynamically. A model can serve as the substratum that allows for the application of knowledge that corresponds to the dynamic form from which it was gleaned. Studies that involve people's tendency to imagine implicit motion in static objects is a good example of the effect of visual models. Freyd, Pantzer, & Cheng (1988), for example, have shown that people will implicitly move objects according to gravity or springiness if a diagram is veridical to the real world but will not do so otherwise. Schwartz and Black (1990) have also found evidence that a proper visual model can elicit a deep knowledge of how objects behave (cf., Kaiser, Proffitt, & Anderson, 1985). The bedrock knowledge of an analog model serves as a generative source of behaviors and possibilities from which the process of rule construction can select and abstract.

Mapping the immediate research and conceptualization into an applied framework appears to be very promising for developing prescriptions for the design and use of manipulatives (Behr, Lesh, Post, & Silver, 1983) and other forms of mediated learning such as those involving computer interfaces (Hollan, Hutchins & Weitzman, 1987). One possible instructional approach that precipitates from this study would go as follows: (a) Have students simulate or envision the basic causal relationships of the target system. (E.g., focus on just two gears.) This will allow them to choose the relevant variables and induce a basic pattern of behavior. It is important that the correct behavior is chosen for modeling. It is not always the case that an analog model is faithful to reality as has been documented with the case of centripetal forces (e.g., Kaiser, et.al., 1985). Care should be taken to tap a level of "intuition" that can be built upon. The manifestation of this intuition need not look like the problem that is to be solved. The steps to understanding circular motion do not begin with circular motion. (b) Provide a set of problems that can be modeled and that vary systematically along the dimension that the desired rule is supposed to capture. (E.g., the number of gears or the oddness and evenness.) This will create the ranges for the input and output domains and reveal more complex patterns. (c) Continually provide examples that promote the generalization of the rule by highlighting the redundancy of separate specific rules. (E.g., change the problem to include a different initial motion or handedness.) This will lead to an abstraction of the parameters and a concomitant distillation of experience that allows more abstract patterns to emerge. The practical implication of this is that several manipulatives may be better than one. (d) Once a rule has been evinced, provide problems that the rule does not adequately cover. This will ensure a complementary relationship between rule and model for the domain of knowledge.

Instruction in formal systems of thought often begins with rules that are the culmination of centuries. This is wrong. The learner must grow into these rules as well. Instruction in only the end product will lead to a brittle and ungenerative understanding.

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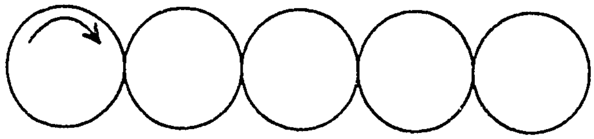
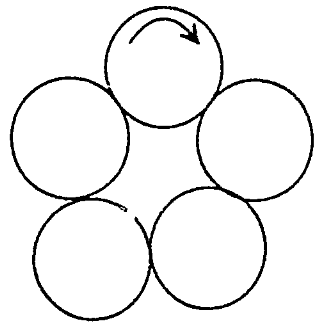
A Visual Representation of the Problem Texts.



"Five gears are arranged in a horizontal line. If you try to turn the gear on the left clockwise, what will the gear on the far right do?"



"Five gears are arranged in a circle so that each gear is touching two other gears. If you try to turn the gear on the top clockwise, what will the gear just to its right do?"

Figure 1

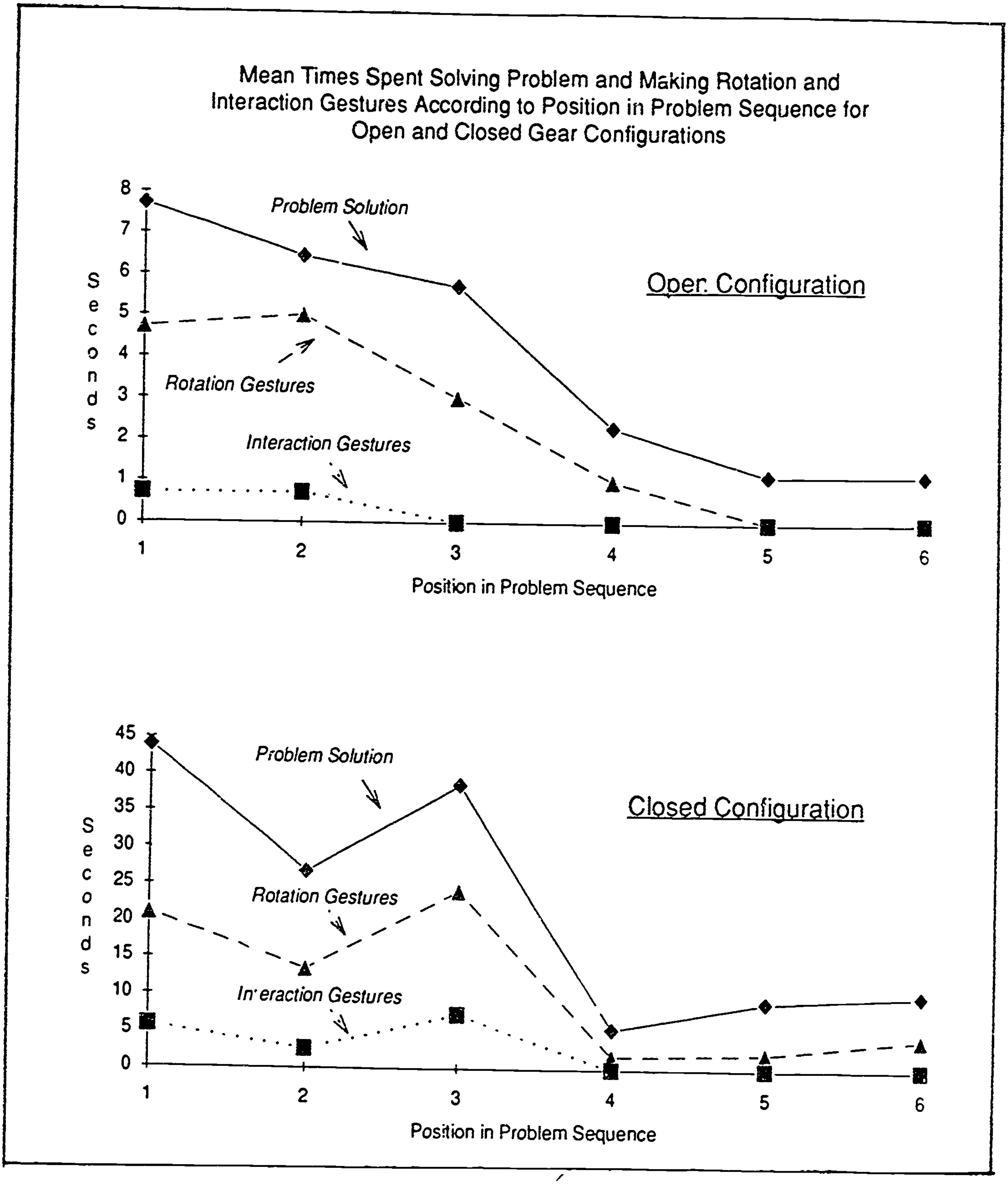

Mean Times Spent Solving Problem and Making Rotation and Interaction Gestures According to Position in Problem Sequence for Open and Closed Gear Configurations
Open Configuration
Problem Solution
Rotation Gestures
Interaction Gestures
Seconds
0
1
2
3
4
5
6
7
8
1
2
3
4
5
6
Position in Problem Sequence
Closed Configuration
Problem Solution
Rotation Gestures
Interaction Gestures
Seconds
0
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
Position in Problem Sequence

Figure 2

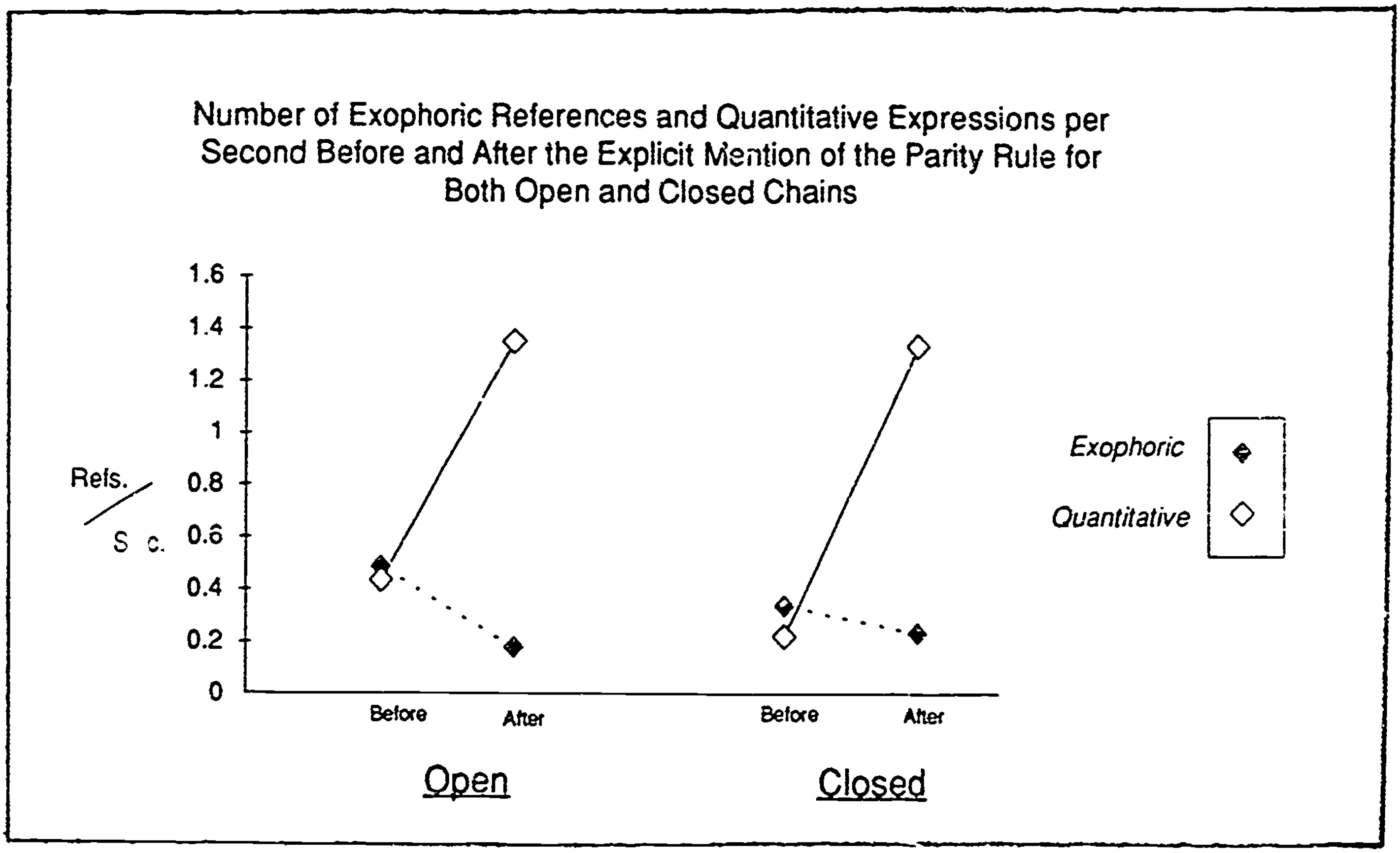

Number of Exophoric References and Quantitative Expressions per Second Before and After the Explicit Mention of the Parity Rule for Both Open and Closed Chains
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Refs./S c.
Before
After
Before
After
Open
Closed
Exophoric
Quantitative

Figure 3